

Wiener Filter-Based Channel Predictor Performance Improvement Using Polynomial Extrapolation

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Abstract-The widely used channel predictor today is Wiener filter-based channel predictor, where the channel is assumed to be random and has time invariant correlation value. Based on the quasi static principle, in the interval of short observation time the channel tends to be deterministic and has time variant correlation value. This makes Wiener filter-based channel predictor can not yield minimum mean squared error. To solve this problem the filter should accommodate the deterministic property and the correlation fluctuation of the channel. This can be achieved by polynomially extrapolating the channel correlation value. This work conducted performance evaluation of conventional Wiener filter and Wiener filter with polynomial extrapolation modification. The performance was evaluated by varying the symbol rates, Doppler frequency and the filter orders. However, the performance of polynomial extrapolation method for the noise was also observed.

Keyword: Wiener filter, polynomial interpolation, quasi static, correlation, channel prediction.

I. INTRODUCTION

The change of the channel condition is utterly affected by the receiver velocity. The higher the receiver velocity is, the faster the change of the channel condition is. When the symbol rate is higher than the rate of the change of the channel condition, the channel condition within one symbol period will be approximately constant (quasi static) [1]. The change of the channel condition within two concatenated symbol period will be linear and can be approximated by linear mathematical function (zeroth or first order polynomial function). The change of the channel condition within more than two concatenated symbol period can be deterministically approximated by many orders of polynomial function, depending on the number of the symbol. This deterministic property makes at the interval of short observation time, the channel correlation value can be deterministically approximated. Moreover, in the interval of short observation time the channel has time variant correlation value, as showed by the observation line in Fig. 1 (black line).

In Fig. 1, the curve shape of 1st observation is different from the curve shape of 2nd observation. This makes the correlation

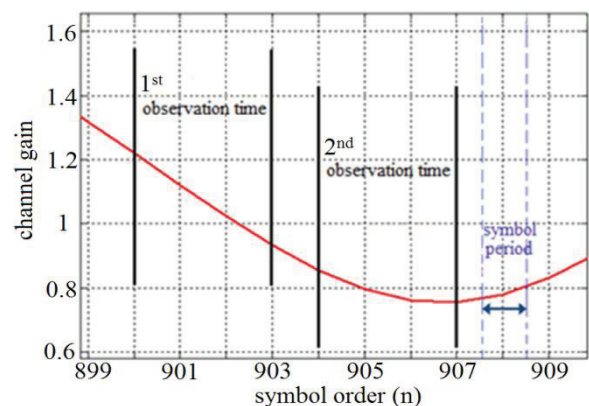


Fig. 1 Example of simulated channel gain change of Rayleigh slow fading in several symbol period.

value at 1st observation is different from the correlation value at 2nd observation.

The widely used channel predictor is Wiener filter-based channel predictor. The predictor assumes that the change of channel condition is random and the channel correlation is time invariant [2]. Deterministic property and the change of channel correlation in the interval of short observation time make the assumptions used by Wiener filter are not totally satisfied, so that the filter will lose its optimality, means that the filter can not yield the minimum mean squared error. Accommodating the deterministic property of the channel, conventional Wiener filter can be modified to maintain its optimality.

The rest of this paper is organized as follows. Section II explains the Rayleigh channel modelling, section III explains the proposed Wiener filter with polynomial extrapolation modification. Simulation results are provided in Section IV.

II. RAYLEIGH CHANNEL MODELLING

The prediction filter works at the receiver, after the initial signal changed into symbol $s(n)$, modulated by the carrier with the frequency of f_c , affected by the channel condition $\beta(n)$ and the noise $V(n)$, as depicted in Fig. 2 [3].

Rayleigh Channel Model

In Rayleigh fading channel, there is no signal delay exceeding the symbol duration. The received symbol can be written as

$$r(n) = s(n)\beta(n)e^{j2\pi f_c t_n} + V(n). \quad (1)$$

The resulted baseband signal after eliminating the carrier can be written as

$$Y(n) = s(n)\beta(n) + v(n). \quad (2)$$

Dividing $Y(n)$ by $s(n)$ will give

$$y(n) = \beta(n) + \tilde{v}(n), \quad (3)$$

where $\beta(n) = \alpha(n)e^{j\phi(n)}$. In (3), $\beta(n)$ is fading fluctuation which is complex Gaussian random variable, $\alpha(n)$ is the amplitude of fading fluctuation which satisfies Rayleigh distribution, $\phi(n)$ is the phase of the received signal and $\tilde{v}(n)$ is noise which is assumed as additive white gaussian noise (AWGN) with zero average [3]-[4].

III. PROPOSED WIENER FILTER WITH POLYNOMIAL EXTRAPOLATION MODIFICATION

In this work, the correlation value of $y(n)$ is assumed as a function of time n and time difference k , $r_y(k, n)$. If $y(n)$ has time variant correlation value, the most suitable correlation used to yield $y(n+p)$ prediction ($\hat{y}(n+p)$) is r_y at $n+p$, since $r_y(k, n+p) \neq r_y(k, n)$. In this work, $r_y(k, n+p)$ is estimated by polynomial extrapolation technique. When $\beta \gg \tilde{v}$, the method is expected to work well.

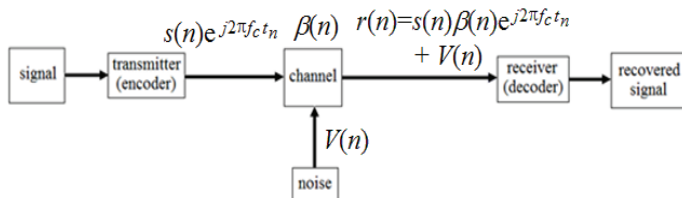


Fig. 2.a Diagram of wireless communication system.

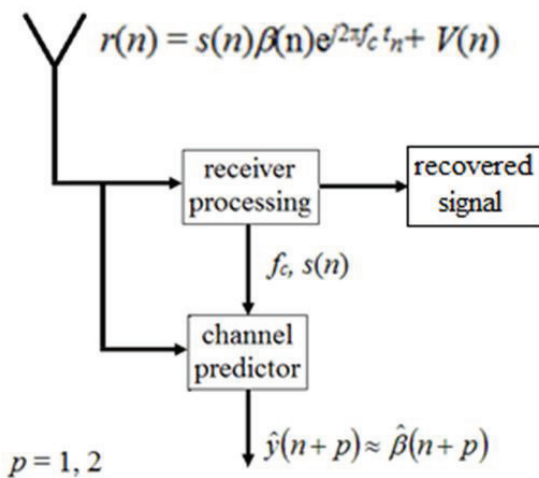


Fig. 2.b The evaluated channel predictor.

A. Basic Arguments of Polinomial Extrapolation

At the slow fading condition, the channel β is only able to slightly change among several concatenated symbols [1]. Whatever the shape of the curve of the channel condition, in a very short time interval (Δt_s) the curve will have approximately constant value, as shown in Fig. 3a. In the Figure, $\beta_{r\Delta t_s}$ is the constant value of the real part of β along Δt_s at time value of t , $\beta_{im\Delta t_s}$ is the constant value of the imaginary part of β along Δt_s at time value of $t - \Delta t$. The correlation between $\beta_{r\Delta t_s}$ and $\beta_{im\Delta t_s}$ is

$$r(\Delta t) = \beta_{r\Delta t_s} \cdot \beta_{im\Delta t_s}. \quad (4)$$

If Δt_s is enlarged into $k(\Delta t_s)$, where k is any arbitrary positive integer, (4) becomes

$$r(\Delta t) = \sum_{i=1}^k ((\beta_{r\Delta t_s})_i \cdot (\beta_{im\Delta t_s})_i) \quad (5)$$

As a continued function, within the Wiener filter operation time interval the channel correlation can be written as

$$r_\beta(\Delta t) = \int_{t_0}^{t_0 + tL} (\beta_r(t) + \beta_{im}(t)j)(\beta_r(t - \Delta t) - \beta_{im}(t - \Delta t)j) dt \quad (6)$$

with t_0 is the initial observation time, tL is the necessary time for Wiener filter with the order of L to take $L+1$ samples of the channel information signal, β_r is the real part of β and β_{im} is the imaginary part of β . Equation (6) can be discretely approximated using (5), where $tL = k(\Delta t_s)$.

Since the change of channel condition has no strong fluctuation between two concatenated symbol, the change of channel correlation within one symbol period will form the simplest curve shape. Fig. 3.a shows an example of the curve of real and imaginary part of β at Rayleigh slow fading. Using (5), the correlation between real and imaginary part of β can be calculated and it is depicted in fig. 3.b and 3.c.

Polynomial interpolation is suitable to estimate the channel correlation change within one or two symbol period, since polynomial interpolation produces the simplest polynomial function connecting all of the observed correlation value. The consequence is that the polinom curve will form the simplest shape between two concatenated correlation value.

The correlation pattern at $n+1$ is highly influenced by the correlation pattern at n and $n-1$, since the channel can not strongly fluctuate within one symbol period (the channel is assumed to be sluggish). Therefore the channel correlation at $n+1$ can be polinomially estimated by observing the channel correlation at $n-1$ and n . The channel correlation at $n+2$ is harder to be estimated. The suitable choice is to take the average value of the previously observed channel correlation as the channel correlation at $n+2$. In this work, polynomial interpolation is termed as polinomial extrapolation since the interpolation is taken for estimating the correlation value outside the filter observation, as will be explained in Fig. 4 until Fig. 6.

B. Initial Calculation of The Correlation Value

This section is focused on the first filter order ($L-1 = 1$). In order to yield $\hat{y}(n+p)$, $\hat{r}_y(p, n+p)$ and $\hat{r}_y(p+1, n+p)$ should be known, where $p = 1, 2$. These correlation value are

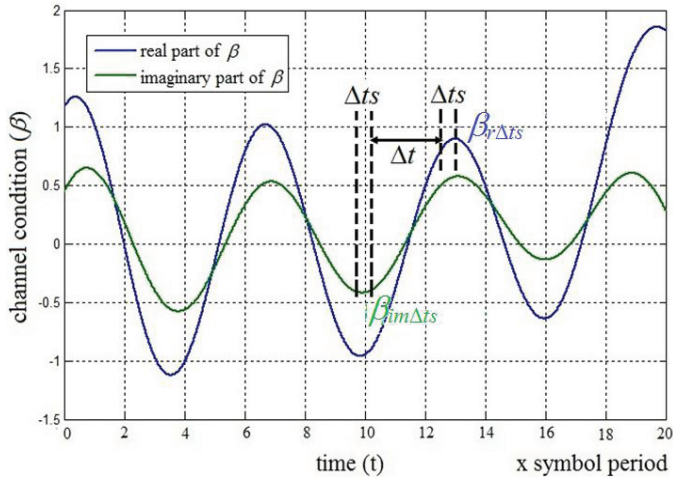


Fig. 3.a Example of simulated Rayleigh slow fading channel.

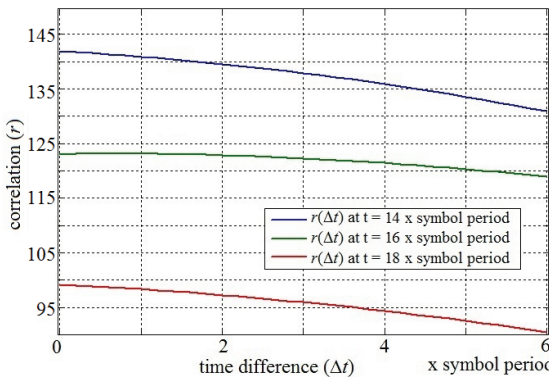


Fig. 3.b Curve shape of correlation of Fig. 3.a as a function of time difference.

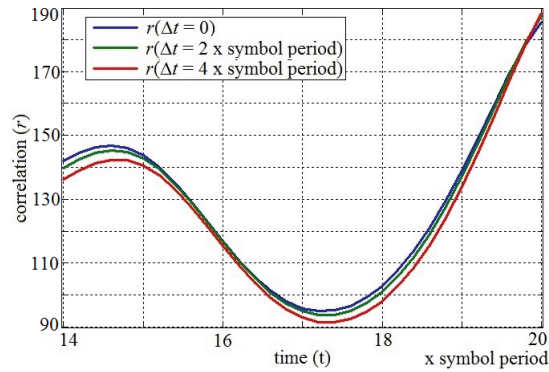


Fig. 3.c Curve shape of correlation of Fig. 3.a as a function of time.

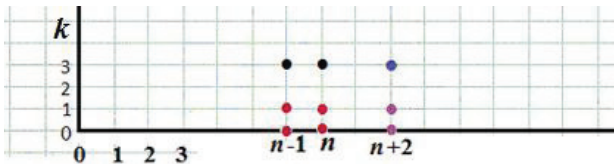


Fig. 4 Used correlation for polynomial extrapolation (first filter order).

estimated using polynomial extrapolation, using the correlation of filter input at $n-1$ and n (red points in Fig. 4).

The initial step is to estimate the correlation values at the purple, blue and black points in Fig. 4, taken as the average

correlation value of two concatenated points from all four red points in Fig. 4. The estimation is taken without using Yule-Walker equation since the channel tends to be deterministic rather than be random.

$$\begin{aligned} \hat{r}_y(0, n+2) &= \frac{r_y(0, n-1) + r_y(0, n)}{2} \\ \hat{r}_y(1, n+2) &= \frac{r_y(1, n-1) + r_y(1, n)}{2} \\ \hat{r}_y(3, n-1) &= \frac{r_y(0, n-1) + r_y(1, n-1)}{2} \\ \hat{r}_y(3, n) &= \frac{r_y(0, n) + r_y(1, n)}{2} \\ \hat{r}_y(3, n+2) &= \frac{\hat{r}_y(0, n+2) + \hat{r}_y(1, n+2)}{2} \end{aligned} \quad (7)$$

C. Polinom Coefficients Calculation

The next step is to find the mathematical equation which connects all correlation values in Fig. 4. The possible mathematical equation is polinomial equation containing time variable n and time difference variable k . At one value of k , the pattern of correlation fluctuation can be represented as

$$r_y(k, n) = C_0(k)n^0 + C_1(k)n^1 + C_2(k)n^2 \quad (8)$$

In (8), the polinom order is $n = 2$ because in Fig. 4 $r_y(k, n)$ are known at three different value of n [2]. $C_0(k)$, $C_1(k)$ and $C_2(k)$ are coefficients which depends on k . Since in Fig. 4 $r_y(k, n)$ are known at three different value of k , $C_0(k)$, $C_1(k)$ and $C_2(k)$ can be written as polinomial function of k with the polinomial order of 2.

$$\begin{aligned} C_0(k) &= K_{20}k^2 + K_{10}k + K_{00}, \\ C_1(k) &= K_{21}k^2 + K_{11}k + K_{01}, \\ C_2(k) &= K_{22}k^2 + K_{12}k + K_{02} \end{aligned} \quad (9)$$

K are the constants representing the dependence of $r_y(n, k)$ on n and k . Combining (8) and (9) will give

$$r_y(k, n) = (K_{20}k^2 + K_{10}k + K_{00})n^0 + (K_{21}k^2 + K_{11}k + K_{01})n^1 + (K_{22}k^2 + K_{12}k + K_{02})n^2 \quad (10)$$

The parts containing k^2n , kn , k^2n^2 , dan kn^2 in (10) should have nonzero value of n and k , since it makes the dependence of $r_y(n, k)$ on n and k will not be perfectly observed. Inserting $k = 0$, (10) becomes

$$r_y(k=0, n) = K_{20}0^2 + K_{10}0 + K_{00} + K_{21}0^2n + K_{11}0n + K_{01}n + K_{22}0^2n^2 + K_{12}0n^2 + K_{02}n^2 \quad (11)$$

In (11), if n is changed, the value of the parts containing n of the equation will not be affected by the change of n . This is not true because all value of K are the representation of the dependence of r_y on n and k . Therefore k should be substituted by a new and nonzero variable, for example $k' = k + 1$. Sustituting k by $k' - 1$, (10) becomes

$$\begin{aligned} r_y(k', n) &= K_{20}(k'-1)^2 + K_{10}(k'-1) + K_{21}(k'-1)^2n \\ &+ K_{11}(k'-1)n + K_{22}(k'-1)^2n^2 \\ &+ K_{12}(k'-1)n^2 + K_{00} + K_{01}n + K_{02}n^2 \end{aligned} \quad (12)$$

$$r_y(k', n) = K'_{20}k'^2 + K'_{10}k' + K'_{00} + K'_{21}k'^2 n + K'_{11}k' n + K'_{01}n + K'_{22}k'^2 n^2 + K'_{12}k' n^2 + K'_{02}n^2 \quad (13)$$

After the variable substitution, (13) becomes

$$r_y(k'=1, n) = K'_{20}1^2 + K'_{10}1 + K'_{00} + K'_{21}1^2 n + K'_{11}1n + K'_{01}n + K'_{22}1^2 n^2 + K'_{12}1n^2 + K'_{02}n^2 \quad (14)$$

It is obvious that all parts containing n of the righthand side of (14) will change as n changes. This shows the dependence of $r_y(n, k)$ on n more accurately. By the same reason, the value of n should not be zero. For simplicity, K' are written as K , just for polinom coefficients generalization.

The longer the filter works, the higher the value of n is. This is not convenient for computation. n can be changed into new variable, $N = (n+2) - (n-x)$, with x are the numbers after n in Fig. 4. Therefore $N = 1, 2, 3, 4$. This is depicted in fig. 5.

Applying (13), after substituting n by N , the equation connecting all of N , k' and correlation values in Fig. 5 can be written as

$$\begin{bmatrix} 1^2 & 1 & 1 & 1^2.1 & 1.1 & 1 & 1^2.1^2 & 1.1^2 & 1^2 \\ 2^2 & 2 & 1 & 2^2.1 & 2.1 & 1 & 2^2.1^2 & 2.1^2 & 1^2 \\ 4^2 & 4 & 1 & 4^2.1 & 4.1 & 1 & 4^2.1^2 & 4.1^2 & 1^2 \\ 1^2 & 1 & 1 & 1^2.2 & 1.2 & 2 & 1^2.2^2 & 1.2^2 & 2^2 \\ 2^2 & 2 & 1 & 2^2.2 & 2.2 & 2 & 2^2.2^2 & 2.2^2 & 2^2 \\ 4^2 & 4 & 1 & 4^2.2 & 3.2 & 2 & 4^2.2^2 & 4.2^2 & 2^2 \\ 1^2 & 1 & 1 & 1^2.4 & 1.4 & 4 & 1^2.4^2 & 1.4^2 & 4^2 \\ 2^2 & 2 & 1 & 2^2.4 & 2.4 & 4 & 2^2.4^2 & 2.4^2 & 4^2 \\ 4^2 & 4 & 1 & 4^2.4 & 4.4 & 4 & 4^2.4^2 & 4.4^2 & 4^2 \end{bmatrix} \begin{bmatrix} K_{20} \\ K_{10} \\ K_{00} \\ K_{21} \\ K_{11} \\ K_{01} \\ K_{22} \\ K_{12} \\ K_{02} \end{bmatrix} = \begin{bmatrix} r_y(1,1) \\ r_y(2,1) \\ \hat{r}_y(4,1) \\ r_y(1,2) \\ r_y(2,2) \\ \hat{r}_y(4,2) \\ \hat{r}_y(1,4) \\ \hat{r}_y(2,4) \\ \hat{r}_y(4,4) \end{bmatrix} \quad (15)$$

Taking the invers of the leftmost part of (15), K_{20} until K_{02} can be known.

D. Extrapolation of The Correlation Value

In order to get $\hat{y}(n+1)$ dan $\hat{y}(n+2)$, the correlation at green points in Fig. 6 should be known [2]. This can be achieved using polinomial extrapolation. Based on (13), after changing n into N , the correlation at the green points in Fig. 6 are given by (16).

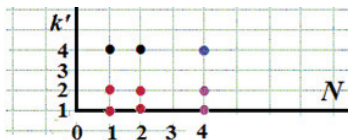


Fig. 5 Fig. 4 after changing k into $k' = k + 1$ and n into $N = (n+2) - (n-x)$, with $x = -1, 0, 1, 2$.

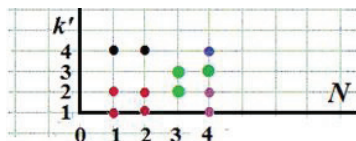


Fig. 6 The green points are the correlations to be estimated through polinomial extrapolation (first filter order).

$$\begin{bmatrix} \hat{r}_y(2,3) \\ \hat{r}_y(3,3) \\ \hat{r}_y(3,4) \end{bmatrix} = \begin{bmatrix} 2^2 & 2 & 1 & 2^2.3 & 2.3 & 3 & 2^2.3^2 & 2.3^2 & 3^2 \\ 3^2 & 3 & 1 & 3^2.3 & 3.3 & 3 & 3^2.3^2 & 3.3^2 & 3^2 \\ 3^2 & 3 & 1 & 3^2.4 & 3.4 & 4 & 3^2.4^2 & 3.4^2 & 4^2 \end{bmatrix} \begin{bmatrix} K_{20} \\ K_{10} \\ K_{00} \\ K_{21} \\ K_{11} \\ K_{01} \\ K_{22} \\ K_{12} \\ K_{02} \end{bmatrix} \quad (16)$$

There is easier way to get $\hat{r}_y(3,4)$. $\hat{r}_y(3,4)$ can be calculated through polinomial interpolation using the correlation values at $N = 4$ in Fig. 6. Based on (13), $r_y(k', N)$ can be written as

$$\begin{aligned} & (K_{20} + K_{21}N + K_{22}N^2)k'^2 + (K_{10} + K_{11}N + K_{12}N^2)k' \\ & + (K_{00} + K_{01}N + K_{02}N^2) = r_y(k', N) \end{aligned} \quad (17)$$

Inserting $N = 4$, every parts in the parentheses in (17) will be constant. Equation (17) can be written as

$$K_2 k'^2 + K_1 k' + K_0 = r_y(k', N) \quad (18)$$

Considering the correlation at $N = 4$ in Fig. 6 and (18) will give

$$\begin{bmatrix} 1^2 & 1 & 1 \\ 2^2 & 2 & 1 \\ 4^2 & 4 & 1 \end{bmatrix} \begin{bmatrix} K_2 \\ K_1 \\ K_0 \end{bmatrix} = \begin{bmatrix} \hat{r}_y(1,4) \\ \hat{r}_y(2,4) \\ \hat{r}_y(4,4) \end{bmatrix} \quad (19)$$

Taking the inverse of the leftmost part of (19), the coefficients K_1 , K_2 and K_3 can be calculated. Inserting $k' = 3$ and $N = 4$ into (18), $\hat{r}_y(3,4)$ can be calculated.

Extrapolation results, as depicted in Fig. 6, are then used to calculate the filter impulse responses $w(l)$ [2].

$$\begin{bmatrix} w(0) \\ w(1) \end{bmatrix} = \begin{bmatrix} \hat{r}_y(1,2+p) & \hat{r}_y^*(2,2+p) \\ \hat{r}_y(2,2+p) & \hat{r}_y(1,2+p) \end{bmatrix}^{-1} \begin{bmatrix} \hat{r}_y(p+1,2+p) \\ \hat{r}_y(p+2,2+p) \end{bmatrix}, p = 1,2 \quad (20)$$

The filter will yield the channel prediction

$$\hat{y}(n+p) \approx \beta(n+p) = \sum_{l=0}^1 w(l)y(n-l), p = 1,2 \quad (21)$$

For second and third filter order, Fig. 4 changes into Fig. 7 and Fig. 8.

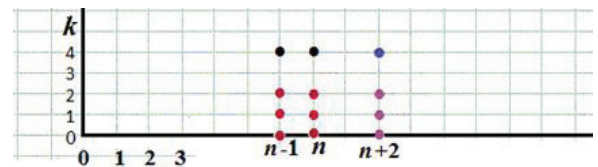


Fig. 7 Used correlation for polinomial extrapolation (second filter order)

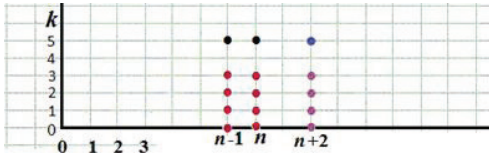


Fig. 8 Used correlation for polinomial extrapolation (third filter order)

IV. SIMULATION RESULTS

This work conducted the simulation of Rayleigh slow fading channel prediction using conventional Wiener filter and Wiener filter with polinomial extrapolation modification. Simulation of noiseless channel prediction was done at varied filter order, varied symbol rate ranging from 20000 symbol/s up to 1020000 symbol/s, carrier frequency of 2000 MHz and receiver velocity of 60 m/s. The results are shown in Fig. 9 until Fig. 12. At the low symbol rate, in this case 20000 symbol/s, polinomial extrapolation method produced less significant accuracy improvement to the conventional Wiener filter. This happens because the lower the symbol rate is, the more rarefied the channel information (β) sampling period is, the more different is the calculated correlation value from the correlation value achieved by (6). This reduces the effectiveness of polinomial extrapolation method. It was also seem that polinomial extrapolation method produced better improvement at the low filter orders and high symbol rate. This indicates that the shorter the observation time interval, the more the assumption used by polinomial extrapolation method be satisfied.

The channel coherent time (T_c) at the carrier frequency of 2000 MHz and receiver velocity of 60 m/s is $(300000000 \text{ m/s}) / (60 \text{ m/s} \times 2000000000 \text{ Hz}) = 1/400 \text{ s}$. At the symbol rate of 320000 symbol/s, the symbol period (T_s) is $1/320000 \text{ s}$. At this condition, T_c/T_s is 800, means that it needs at least 800 of β samples to exploit T_c . Therefore, it can be known that at the symbol rate of 320000 symbol/s or more, 1st up to 8th filter order, the change of the channel condition is linear. At the symbol rate of 320000 symbol/s or higher, $\beta(n+2)$ prediction using fourth filter order of polinomial extrapolation method produced very low error percentage since it produced linear output to the input.

Simulation of Rayleigh channel prediction with 10 dB of AWGN was done at varied filter order, varied Doppler frequency ranging from 360 Hz up to 440 Hz and symbol rate of 520000 symbol/s. The results are shown in Fig. 13 and Fig. 14. It seems that the error percentage of $\beta(n+1)$ prediction using polinomial extrapolation method is very fluctuative because of the distortion of the channel correlation due to AWGN. Since the used correlations for $\beta(n+1)$ prediction are totally determined by the polinomial function, any little change in the channel correlation will cause significant change in the polinomial function value. The higher the filter order is, the more distorted the channel correlation is, the more is the prediction error percentage. Fig. 13 and fig. 14 show that $\beta(n+2)$ prediction error percentage was not so fluctuative, since the used correlation for $\beta(n+2)$ prediction is the average of the previously observed correlation. Since AWGN has average

value of zero, the average of the distortion of channel correlation will also be zero. This makes the used correlations for $\beta(n+2)$ prediction be less distorted than the used correlations for $\beta(n+1)$ prediction. The figures also show that

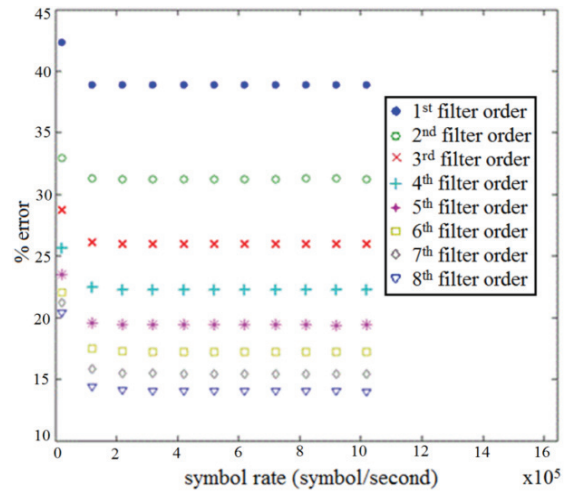


Fig. 9 $\beta(n+1)$ prediction error percentage of polinomial extrapolation method for varied filter order and varied symbol rate at receiver velocity of 60 m/s and carrier frequency of 2000 MHz.

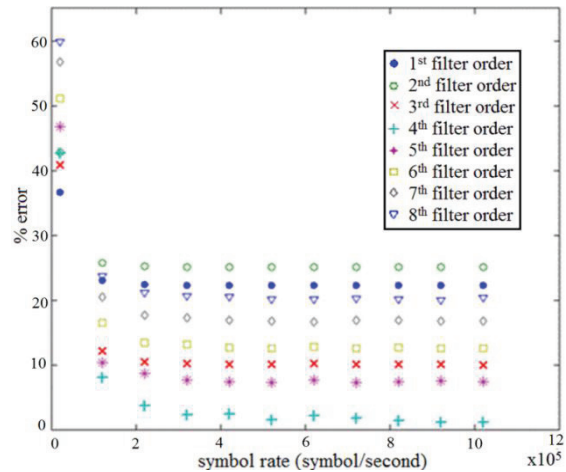


Fig. 10 $\beta(n+2)$ prediction error percentage of polinomial extrapolation method for varied filter order and varied symbol rate at receiver velocity of 60 m/s and carrier frequency of 2000 MHz.

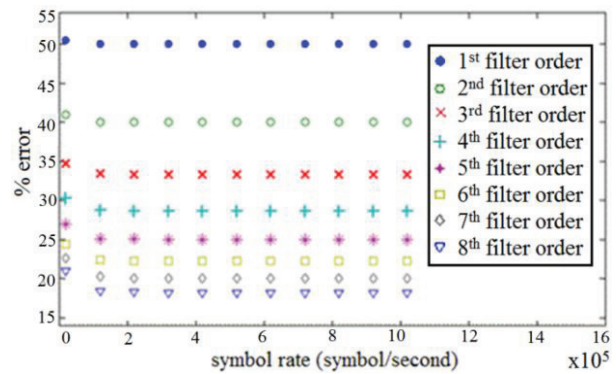


Fig. 11 $\beta(n+1)$ prediction error percentage of conventional Wiener filter for varied filter order and varied symbol rate at receiver velocity of 60 m/s and carrier frequency of 2000 MHz.

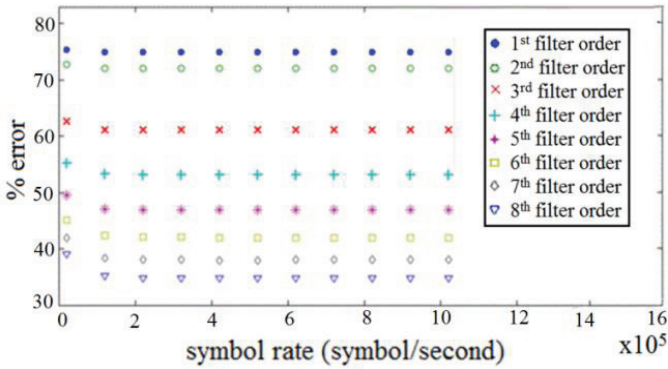


Fig. 12 $\beta(n+2)$ prediction error percentage of conventional Wiener filter for varied filter order and varied symbol rate at receiver velocity of 60 m/s and carrier frequency of 2000 MHz.

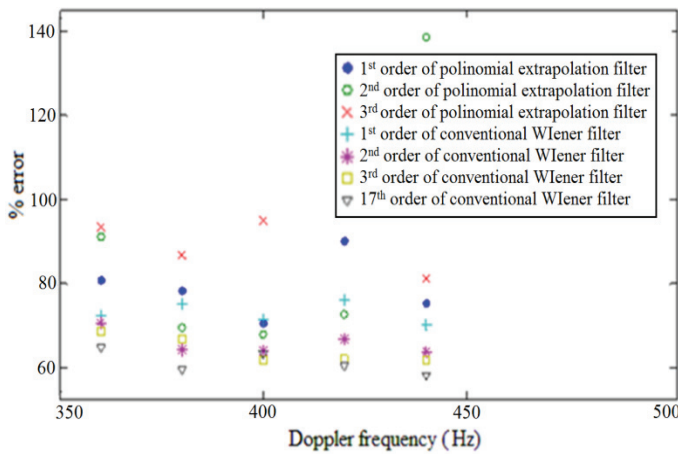


Fig. 13 $\beta(n+1)$ prediction error percentage for varied filter order and varied Doppler frequency at symbol rate of 520000 symbol/s with SNR of 10 dB.

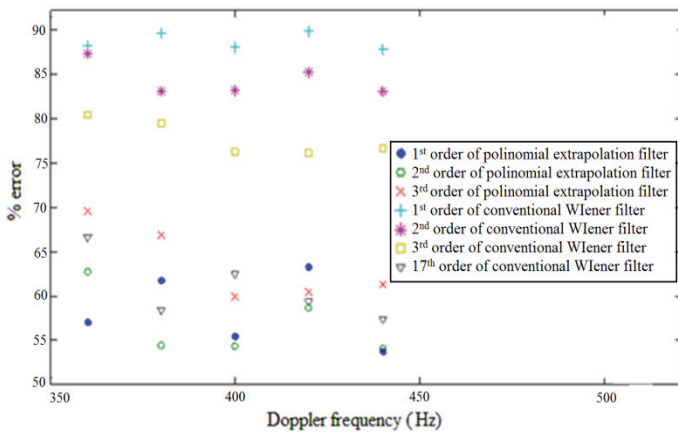


Fig. 14 $\beta(n+2)$ prediction error percentage for varied filter order and varied Doppler frequency at symbol rate of 520000 symbol/s with SNR of 10 dB.

$\beta(n+2)$ prediction using 2nd order of polynomial extrapolation filter produces lower prediction error percentage which can not be achieved by many orders of conventional Wiener filter. It seems that polynomial extrapolation method for the first and second filter order produced the least error percentage. This is because the first and second filter order get the minimum noise influence than the other filter orders do.

V. CONCLUSION

The simulation results show that in the interval of short observation time, the assumptions that the channel is deterministic and has time variant correlation value are more satisfied than the assumptions that the channel is random and has time invariant correlation value. This was indicated from the noiseless channel prediction error percentage achieved by polynomial extrapolation method which was lower than the prediction error percentage produced by conventional Wiener filter, especially at high symbol rate. At high symbol rate, the accuracy improvement made by polynomial extrapolation method was tightly depend on the filter order and very slightly depend on the comparison between channel coherent time and symbol period (T_c/T_s). At this condition, the higher the filter order is, the lower the accuracy improvement made by the polynomial extrapolation method.

Polynomial extrapolation method has $\beta(n+1)$ prediction accuracy which more sensitive to AWGN than $\beta(n+2)$ prediction accuracy does, especially at high symbol rate. In the influence of 10 dB of AWGN and at high symbol rate, $\beta(n+2)$ prediction using polynomial extrapolation method produces less prediction error percentage than $\beta(n+1)$ and $\beta(n+2)$ prediction using many orders of conventional Wiener filter.

ACKNOWLEDGMENT

The authors are grateful to Suhartono Tjondronegoro for his continuous encouragement and stimulus during preparation of this paper. Thank you for transferring a lot of knowledge and having interesting discussions.

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