# High and Low Rank MIMO Channel Capacity on MIMO-Wireless Communication Systems

Rina Pudji Astuti[a], Tati L.R Mengko[b], Sugihartono[c], Andriyan B. Suksmono[d]

[a,b,c,d] ITB, Bandung, Indonesia

**Abstract.**

The impact of a rich scattering environment and channel knowledge in channel performance will determine the MIMO-wireless systems capacity. It is also depend on MIMO inter antennas elements correlation. To achieve a high MIMO system capacity, the orthogonality (un-correlation) of signals from MIMO antenna elements must be guaranteed. The condition will cause a change on MIMO channel matrix, $\mathbf{H}$, become high-rank or low-rank channels. The high-rank or low-rank channels will decide MIMO-wireless systems capacity.

In general, both of the MIMO systems and the MIMO-STBC systems with the no channel knowledge at the receiver have more capacity than the systems with the channel knowledge at the receiver. However with the small S/N , the systems with the channel knowledge at the receiver have better capacity performance than the other.

Both of the MIMO channel and the MIMO-STBC channel with the same number of receive antennas and the large S/N, double of the number of transmit antennas has a large increase capacity performance. In high-rank and low rank correlation**,** the MIMO channel has a better capacity performance than the MIMO with STBC.

**Key-words:** MIMO, STBC, channel capacity, high and low rank

## 1 Introduction

The steep progress of wireless communication services has impact on the need of a huge bandwidth to support new wireless application or services. On the other hand, frequency bandwidth is the limited resources. So, a system which has high frequency bandwidth usage efficiency is needed to increase an average capacity. MIMO (multiple inputs and multiple outputs) is one of the solutions to handle the condition.

The basic idea of MIMO systems is *space time signal processing.* The technique utilizes combination of signal processing in time dimension, which is a natural dimension of digital communication data, and space dimension which is a usage of array antenna in space distribution. The large spectral efficiencies associated with MIMO channels are based on multipath propagation with a rich scattering environment provides independent transmission path from each transmit antenna. On the other hand MIMO takes advantage of random fading [Foshini'96,'98] for increasing transfer rates.

Channel capacity characteristic of MIMO system is depend on readiness of channel model and estimation in both of transmission and receiver side. The MIMO channel capacity is also depend on MIMO inter antennas elements correlation. To achieve a high MIMO system capacity, the orthogonality (un-correlation) of signals from MIMO antenna elements must be guaranteed. The condition will cause a change on MIMO channel matrix, $\mathbf{H}$, become high-rank or low-rank channels. The high-rank or low-rank channels will determine MIMO-wireless systems capacity.

Therefore, our objective is to determine the impact of a rich scattering environment and channel knowledge in channel performance which is indicated in wireless systems capacity. The focus of this paper is to compare the channel performance of MIMO system with the MIMO-STBC system.

This paper is organized as follows. In section 2, MIMO system model and channel model are described. In this section, MIMO-wireless systems with space time block code (STBC) is applied in a rich scattering environment, such as Rayleigh and Rician fading models. Next, section 3 describes information theoretic capacity of MIMO systems. The capacity of MIMO systems includes MIMO with and without STBC system. Next, section 4 describes the results. In this section is discussed the effect of parameter propagation and channel knowledge changes in MIMO systems capacity. Finally, section 5 summarizes altogether the capacity results.

# 2 MIMO system model

Multiple input and multiple output (MIMO) systems, employing several transmit and receive antenna at both ends, are able to increase in capacity compared to traditional single antenna systems. However, this increasing in capacity is dependent upon the fact that the channels from a transmitter to a receiver follow independent path. If a severe correlations present at the transmitter and/or the receiver side, for example,_the capacity of the MIMO systems can be shown to degrade[1].

In order to make the availability of independent channel, we consider to applied space time block code (STBC) such as an inner coding. STBC, however, are not designed to provide significantly coding gain. Hence, powerful outer codes, combining of convolution code and block interleaving, can be concatenated with STBC to have a required coding gain. STBC are applied according to H-BLAST model (as horizontal encoder) which are fixed to each antennas arm.

The channel model of the MIMO systems can be illustrated as follow:

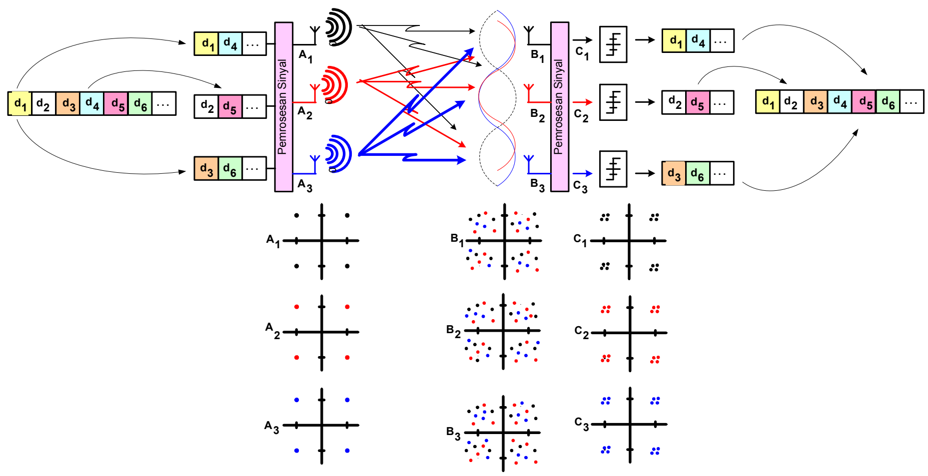

Figure 1. Spatial multiplexing scheme of the MIMO systems with spectral efficiency

A high data bit streams is decomposed into 3 independent of 1/3 rate bit sequences. Then, the data streams are transmitted simultaneously using multiple antennas with spectral efficiency, thus consuming 1/3 of the nominal spectrum. At the receiver, the mixing channel matrix is identified by training symbols. Later, the individual bit streams are separated and estimated. This occurs in the same way as three unknowns are resolved from a linear system of three equations.

The separation of MIMO Channels is possible only if the equations are independent which can be interpreted by each antenna a sufficiently different channel. Wherein case the bit streams can be detected and merged together to obtain the original high data rate signal.

The received signal of the MIMO system at $n^{th}$ antenna receiver from $m^{th}$ antenna transmitter with input signal, $s_m(t)$, is as follow:

$$y_n(t) = \sum_{m=1}^{M_T} h_{m,n}(\tau,t) * s_m(t) \tag{1}$$

Where:

$n=1, 2, \ldots, N_R$

$s_m(t)$ : is input signal of $m^{th}$ antenna transmitter, $T_X$

$h_{m,n}(\tau,t)$ : is channel impulse response of $m^{th}$ antenna transmitter to $n^{th}$ antenna receiver which be offered by propagation channel characteristic, pulse shaping at the transmitter and matched filter at the receiver.

MIMO-wireless channel model in flat fading condition, where $M_T$ antennas in transmitter and $N_R$ antennas in receiver is a matrix $\mathbf{H}$ with $N_R \mathrm{x} M_T$ elements, has signal equation in receiver as follow:

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} \tag{2}$$

Where: $\mathbf{y}$ is receive signal vector, $N_R$ x 1

$s = [s_1 s_2 \ldots s_{M_T}]$ is transmit signal vector, $M_T$ x 1

$\mathbf{H}$ is MIMO channel matrix, $N_R \mathrm{x} M_T$

$\mathbf{n}$ is *additive white Gaussian noise vector,* $N_R$ *x 1*

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M_T} \\ h_{21} & h_{22} & \cdots & h_{2M_T} \\ \vdots & \ddots & \ddots & \vdots \\ h_{N_R} & \cdots & \cdots & h_{N_R M_T} \end{bmatrix} \tag{3}$$

Early MIMO-wireless system model with space time block code (STBC) using two transmit antennas and one receive antenna is suggested by Alamouti[6]. This scheme supports maximum-likelihood (ML) detection based only on linear processing at the receiver. Furthermore, Vahid Tarokh [6] develops the system for general configuration systems. In this scheme, a number of code symbols equal to the number of $T_X$ antennas are generated and transmitted simultaneously, one symbol from each antenna. These symbols are generated by the space time encoder, the diversity gain and/or the coding gain is maximized.

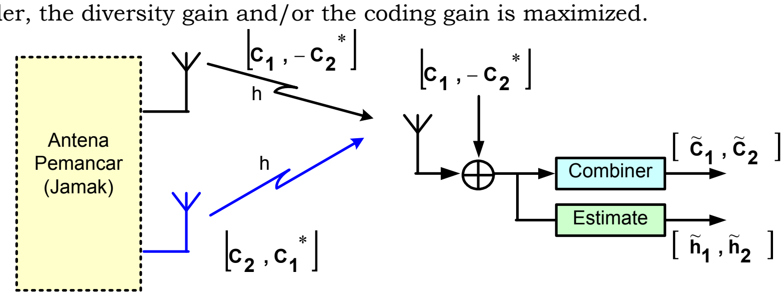


Figure 2. MIMO-STBC system of Alamouti scheme

The Alamouti STBC models, the input symbols to the space time block encoder are divided into groups of two symbols each. At a given symbol period, the two symbols in each group $(c_1, c_2)$ are transmitted simultaneously from the two antennas. The signal transmitted from antenna 1 is $c_1$ and the signal transmitted from antenna 2 is $c_2$. In next symbol period, the signal $-c_2^*$ is transmitted from antenna 1 and the signal $c_1^*$ is transmitted from antenna 2. Let $h_1$ and $h_2$ be the channel from the first and second $T_X$ antennas to the $R_X$ antenna, respectively [1]. It is assumed a receiver with a single $R_X$ antenna, and denotes the received signal over two consecutive symbol periods as $x_1$ and $x_2$. The received signals can be expressed as follow:

$$x_1 = h_1 c_1 + h_2 c_2 + n_1 \quad (4)$$

$$x_2 = -h_1 c_2^* + h_2 c_1^* + n_2 \quad (5)$$

The received signals can be rewritten in a matrix form as:

$$\mathbf{x} = \mathbf{c}.\mathbf{H} + \mathbf{n} \quad (6)$$

Where:

$$\mathbf{x} = [x_1 \quad x_2]^T \quad (7)$$

$$\mathbf{c} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \quad (8)$$

The individual rows correspond to time diversity and the individual columns correspond to space (antenna) diversity.

$$\mathbf{H} = [h_1 \quad h_2]^T \quad (9)$$

$$\mathbf{n} = [n_1 \quad n_2]^T \quad (10)$$

Equation (6)-(8) can be rewritten as follow:

$$\mathbf{x} = \mathbf{H}.\mathbf{c} + \mathbf{n} \quad (11)$$

$$\mathbf{x} = \left[x_1 \quad x_2^*\right]^T \quad (12)$$

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \quad (13)$$

$$\mathbf{c} = [c_1 \quad c_2]^T \quad (14)$$

$$\mathbf{n} = \left[n_1 \quad n_2^*\right]^T \quad (15)$$

A virtual MIMO matrix, $\mathbf{H}$, with space (columns) and time (rows) dimensions, is not to be confused with the purely spatial MIMO channel matrix in previous sections.

# 3 Information theoretic capacity of MIMO systems

## 3.1 Introduction

The spectral efficiencies related with MIMO channels are based on a rich scattering environment provides independent transmission paths each transmit antenna to each receive antenna. The capacity linearly with min $(M_T, N_R)$ relative to a system with just one transmit antenna and one receive antenna. This capacity increase requires a scattering environment such that the matrix of channel gains has full rank and independent entries and that perfect estimates of these gain are available at the transmitter and the receiver [2].

MIMO channel capacity depends greatly on the statistical properties and antenna correlations of the channel. Antenna correlation varies drastically as a function of the scattering environment, the distance between transmitter and receiver, the antenna configurations, and the Doppler spreads [2]. In real-world cases, correlated channel path gains occur.

When there is a rich scattering environment and when $T_X$ and $R_X$ arrays are relatively **near one another**, **high rank MIMO channels** occur. It is occur when there is **little correlation** among the path gains. MIMO channels have a **diversity gain** defined by the rank of $\mathbf{HH}^*$ which is achieve the maximum diversity if $rank(HH^*) = \min(M_T, N_R)$. The upper limit for the capacity of MIMO channels and the maximum diversity gain is represented by the orthogonal channel case.

The **low rank MIMO channel** is equivalent to a single antenna channel with the same total power. It is occur under **scatter-free** or **long-distance links**. It is occur when there is **strong correlation** between the channel path gains. The correlation characteristics decide the rank of $\mathbf{HH}^*$, which in turn determines the diversity advantage. A full correlated $\mathbf{H}$ matrix is a scaled version of the one's matrix with dimensions $(M_T, N_R)$ and provides no diversity gain over the single antenna case.

On MIMO channel capacity in Shannon theoretic sense, the Shannon (ergodic) capacity of a single user time invariant channel is defined as the maximum mutual information between the channel input and output. This maximum mutual information is shown by Shannon's capacity theorem to be the maximum data rate that can be transmitted over the channel with random small error probability. While the channel is time-varying channel capacity has multiple definitions, depending on what is channel knowledge about the channel distribution or its state at the transmitter and/or receiver and whether the capacity is measured based on minimum rate or maintaining a constant fixed or averaging the rate over all channel distributions/states.

## 3.2 MIMO fading channel capacity

### 3.2.1 Information theoretic definition

The entropy of a random variable is a measure of uncertainty of the random variable. It is a measure of the amount of information required on the average to describe the random variable [4].

The mutual information can be described as the reduction in the uncertainty of one random variable due to the knowledge of the other [2]. It will depend on the properties of the wireless channel used to express information from the transmitter to the receiver. With $h_{de}(.)$ denoting differential entropy (entropy of a continuous random variable), the mutual information can be expressed as

$$I(\mathbf{S};\mathbf{Y}) = h_{de}(\mathbf{Y}) - h_{de}(\mathbf{Y}\mid\mathbf{S}) \quad (16)$$

$$= h_{de}(\mathbf{Y}) - h_{de}(\mathbf{HS}+\mathbf{N}\mid\mathbf{S})$$

$$= h_{de}(\mathbf{Y}) - h_{de}(\mathbf{N}\mid\mathbf{S})$$

$$= h_{de}(\mathbf{Y}) - h_{de}(\mathbf{N}) \quad (17)$$

It will be assumed that $\mathbf{N} \sim N(0,\mathbf{K}^n)$, where $\mathbf{K^n} = E\{\mathbf{NN^H}\}$ is the noise covariance matrix.

Because the normal distribution maximizes the entropy over all distributions with the same covariance (i.e. the power constraint), the mutual information is maximized when $\mathbf{Y}$ represents a multivariate Gaussian random variable, i.e. $\mathbf{Y} = N(0,\mathbf{K}^y)$. With the assumption that $\mathbf{S}$ and $\mathbf{N}$ are uncorrelated, the received covariance matrix of the desired signal can be expressed as follow:

$$\mathbf{K^y} = E\{\mathbf{YY^Y}\} = E\{(\mathbf{HS}+\mathbf{N})(\mathbf{HS}+\mathbf{N})^H\} \quad (18)$$

$$= \mathbf{HK}^S\mathbf{H}^H + \mathbf{K}^n \quad (19)$$

where $\mathbf{K}^S = E\{\mathbf{SS}^H\}$ (20)

### 3.2.2 Capacity of MIMO channel

When perfect channel knowledge at the receiver by assuming maximum ratio combining at the receiver and the transmitter has no knowledge of the channel, it is optimal to consistently distribute the available power $P_T$ between the transmit antennas [4], i.e. $\mathbf{K}^S = \frac{P_T}{M_T}\mathbf{I}_{n_S}$. We assume that the noise is uncorrelated among branches, the noise covariance matrix $\mathbf{K}^n = \sigma_n^2\mathbf{I}_{n_R}$. Therefore, the MIMO fading channel capacity can be written as:

$$C = h_{de}(\mathbf{Y}) - h_{de}(\mathbf{N}) \quad (21)$$

$$= \log_2[\det(\pi e(\mathbf{HK}^S\mathbf{H}^H + \mathbf{K}^n))] - \log_2[\det(\pi e\mathbf{K}^n)]$$

$$= \log_2[\det(\mathbf{HK}^S\mathbf{H}^H\mathbf{K}^n)] - \log_2[\det\mathbf{K}^n]$$

$$= \log_2[\det(\mathbf{HK}^S\mathbf{H}^H + \mathbf{K}^n)(\mathbf{K}^n)^{-1})]$$

$$= \log_2[\det(\mathbf{HK}^S\mathbf{H}^H(\mathbf{K}^n)^{-1} + \mathbf{I}_{n_R})]$$

$$= \log_2\left[\det\left(\mathbf{I}_{N_R} + (\mathbf{K}^n)^{-1}\mathbf{HK}^S\mathbf{H}^H\right)\right]$$

$$= \log_2\left[\det\left(\mathbf{I}_{N_R} + \frac{P_T}{\sigma_n^2 M_T}\mathbf{HH}^H\right)\right] \quad (22)$$

If we assume that $\mathbf{H}$ is a random process, we can identify the channel at the receiver by using a training sequence with assuming the training sequence does not cost any capacity. By condition it is no channel knowledge at the receiver, the MIMO channel capacity takes expectation over channel instantiations, as follow:

$$C = E\left\{\log_2 \det\left(I_{N_R} + \frac{P_T}{\sigma_n^2 M_T}\mathbf{HH}^H\right)\right\} \tag{23}$$

## 3.2.3 MIMO with STBC fading channel capacity

For a 2 x 2 MIMO channel, accompany equation (6), the received signal can be expressed as

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} + \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \tag{24}$$

Which can be reorganized and written as[3][4] :

$$\underbrace{\begin{bmatrix} x_{11} \\ x_{21}^* \\ x_{12} \\ x_{22}^* \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \\ h_{12} & h_{22} \\ h_{22}^* & -h_{12}^* \end{bmatrix}}_{\mathcal{H}} \underbrace{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}_{c} + \underbrace{\begin{bmatrix} n_{11} \\ n_{21}^* \\ n_{12} \\ n_{22}^* \end{bmatrix}}_{n} \tag{25}$$

$x_{11}$ and $x_{12}$ represent the received symbols at the first transmit antenna and the second transmit antenna at time index $t$ and also $x_{21}$ and $x_{22}$ signify the received symbols the first transmit antenna and the second transmit antenna at time index $t + T_S$.

With assuming that the MIMO-STBC channel is perfect channel knowledge at the receiver, example with matched filtering at the receiver, the received signal after matched filtering may be expressed as:

$$\mathbf{y} = \mathcal{H}^H \mathbf{x} \tag{26}$$

$$= \mathcal{H}^H \mathcal{H}\mathbf{c} + \mathcal{H}^H \mathbf{n}$$

$$= \|\mathbf{H}\|_F^2 \mathbf{c} + \mathcal{H}^H \mathbf{n} \tag{27}$$

Where:

$$\mathcal{H}^H \mathcal{H} = \begin{bmatrix} h_{11}^* & h_{21} & h_{12}^* & h_{22} \\ h_{21}^* & -h_{11} & h_{22}^* & -h_{12} \end{bmatrix}\begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \\ h_{12} & h_{22} \\ h_{22}^* & -h_{12}^* \end{bmatrix} \tag{28}$$

$$= \begin{bmatrix} |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 \end{bmatrix}$$

$$= \|\mathbf{H}\|_F^2 .\mathbf{I}_2 \tag{29}$$

$\|\mathbf{H}\|_F^2$ is the squared Frobenius norm of the matrix $\mathbf{H}$.

The received signal after matched filtering can be written individually as:

$$y_1 = \|\mathbf{H}\|_F^2 c_1 + \mathcal{H}^H \mathbf{n} \tag{30}$$

$$y_2 = \|\mathbf{H}\|_F^2 c_2 + \mathcal{H}^H \mathbf{n} \tag{31}$$

In general, it can be written as:

$$y_l = \|\mathbf{H}\|_F^2 c_l + \mathcal{H}^H \mathbf{n} \tag{32}$$

Then, the capacity of a MIMO fading channel using STBC can be written as:

$$C = \frac{S}{T}.\log_2\left(1 + \frac{P_T}{\sigma_n^2 M_T}\|\mathbf{H}\|_F^2\right) \tag{33}$$

Where the symbols, $S$, of STBC system are transmitted in the time slots, $T$. The $\frac{S}{T}$ denotes the rate of the STBC.

## 4 Results

Added analysis of the MIMO channel capacity is possible by diagonalizing the product matrix $\mathbf{HH}^H$ also by eigenvalue decomposition or singular value decomposition. Eigenvalue decomposition of matrix product is $\mathbf{HH}^H = \mathbf{E}\Lambda\mathbf{E}^H$, where $\mathbf{E}$ is the eigenvector matrix with orthonormal columns and $\Lambda$ is a diagonal matrix with the eigenvalues on the main diagonal. While singular value decomposition of the channel matrix is $\mathbf{H} = \mathbf{U}\sum\mathbf{V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices of left and right singular vectors respectively. $\sum$ is a diagonal matrix with singular values on the main diagonal.

With the singular value decomposition approach, the capacity of the MIMO system, equation (22) may be expressed as follow:

$$C = \log_2\left[\det\left(\mathbf{I}_{N_R} + \frac{P_T}{\sigma_n^2 M_T}\mathbf{U}\sum\sum^H\mathbf{U}^H\right)\right] \tag{34}$$

$$= \log_2\left[\det\left(\mathbf{I}_{N_R} + \frac{P_T}{\sigma_n^2 M_T}\mathbf{U}^H\mathbf{U}\sum^2\right)\right]$$

$$= \log_2\left[\det\left(\mathbf{I}_{N_R} + \frac{P_T}{\sigma_n^2 M_T}\sum^2\right)\right]$$

$$= \log_2\left[\left(1+\frac{P_T}{\sigma_n^2 M_T}\sigma_1^2\right)\left(1+\frac{P_T}{\sigma_n^2 M_T}\sigma_2^2\right)\cdots\left(1+\frac{P_T}{\sigma_n^2 M_T}\sigma_k^2\right)\right] \quad (35)$$

$$= \sum_{l=1}^{k}\log_2\left(1+\frac{P_T}{\sigma_n^2 M_T}\sigma_l^2\right) \quad (36)$$

Otherwise, with an eigenvalue decomposition of the matrix product $\mathbf{HH}^H$, the MIMO system capacity may be written as follow:

$$C = \sum_{l=1}^{k}\log_2\left(1+\frac{P_T}{\sigma_n^2 M_T}\lambda_l\right) \quad (37)$$

Where :

$$k = rank(\mathbf{H}) \le \min(M_T, N_R) \quad (38)$$

$\sum$ is a real matrix

$\det(\mathbf{I}_{AB} + \mathbf{AB}) = \det(\mathbf{I}_{BA} + \mathbf{BA})$

$\lambda_l$ are the eigenvalue of matrix $\Lambda$.

Although STBC provide full diversity over the coherent, flat fading channel, at a low computational cost. It can be shown that they incur a loss in capacity because the convert the matrix channel into a scalar AWGN channel whose capacity is smaller than the true channel capacity. The MIMO system capacity, equation (34), may be written as follow:

$$C = \log_2\left(1+P\sum_{l=1}^{k}\sigma_l^2 + P^2\sum_{l_1 \ne l_2}^{l_1<l_2}\sigma_{l_1}^2\sigma_{l_2}^2 + \cdots + P^k\prod_{l=1}^{k}\sigma_l^2\right) \quad (39)$$

$$= \log_2\left(1+P\|\mathbf{H}\|_F^2 + P^2\sum_{l_1 \ne l_2}^{l_1<l_2}\sigma_{l_1}^2\sigma_{l_2}^2 + \cdots + P^k\prod_{l=1}^{k}\sigma_l^2\right) \quad (40)$$

$$\ge \log_2(1+P\|\mathbf{H}\|_F^2)$$

$$\ge \frac{S}{T}\log_2\left(1+P\|\mathbf{H}\|_F^2\right) \quad (41)$$

(the MIMO-STBC system capacity)

Where: $$P = \frac{P_T}{\sigma_n^2 M_T} \quad (42)$$

The MIMO system capacity above is explicitly equal or large than the MIMO-STBC system capacity. The capacity difference is a function of channel singular values. This can used to decide under which conditions STBC is optimal in terms of capacity.

In practice, the realization of high MIMO capacity is responsive not only to the fading correlation between individual antennas but also to the rank performance of the channel. High rank behavior has been heavily linked to the existence of a rich scattering environment and a little correlation between the channel path gains, for example in Rayleigh fading model. The MIMO channels have a diversity gain defined by the rank of $\mathbf{HH}^*$. The maximum achievable diversity gain is $rank(\mathbf{HH}^*) = \min(M_T, N_R)$. The maximum diversity gain and the upper limit for the capacity of MIMO channels is represented by the orthogonal channel gain case. With assuming $M_T$ columns of $\mathbf{H}$ are orthogonal and the entries of $\mathbf{H}$ are normalized to unit power, so that the eigenvalues of $\mathbf{HH}^H$ are $\lambda_l = N_R$ for $l = 1,2,\cdots,M_T$. The capacity of the high-rank MIMO channel can be rewritten from equation (36) and (37) as:

$$C_{high\_rank} = \sum_{l=1}^{k=\min(M_T,N_R)} \log_2\left(1+\frac{P_T}{\sigma_n^2 M_T}\lambda_l\right) \quad (43)$$

$$\approx \underbrace{\min(M_T,N_R)}_{\substack{array\_capacity \\ advantage}} \log_2 \underbrace{\left(1+\frac{P_T}{\sigma_n^2 M_T}N_R\right)}_{\substack{receiver\_antenna \\ SNR\_advantage}} \quad (44)$$

On the other hand, Low rank performance has been closely linked to the being of a poor scattering environment and a strong correlation between the channel path gains, for example in Rician fading model with large K factor. The correlation characteristics determine the rank of $\mathbf{HH}^H$, which in revolve determines the diversity advantage. A full correlated $\mathbf{H}$ matrix provides no diversity gain over the single antenna case and gives the all one's matrix with dimension $M_T x N_R$. With assuming high correlation, all gains $h_{ij}$ are approximately equal, and $\mathbf{H}$, a multiple of the all-one matrix, has a single non zero singular value

$$\lambda_l = N_R \sum_{l=1}^{M_T} E(h_{ll}^H h_{ll}) \approx M_T N_R \quad (45)$$

The capacity of the low-rank MIMO channel can be expressed as :

$$C_{low\_rank} = \sum_{l=1}^{k=\min(M_T,N_R)} \log_2\left(1+\frac{P_T}{\sigma_n^2 M_T}\lambda_l\right) \quad (46)$$

$$\approx \log_2\left(1+\frac{P_T}{\sigma_n^2}N_R\right) \quad (47)$$

The low-rank MIMO channel behaves like a point to point channel or a single antenna channel with $N_R$ times the received signal power due to the antenna array. It is achieved by simple maximum-ratio combining at the receiver.

If Low-SNR MIMO channel, $\left(P_T / \sigma_n^2\right)$, is low with a Taylor series approximation of $\log(1+x) \approx x$ for small values of $x$, both of the capacity of the high-rank MIMO channel for small SNR, $C_{high\_SNR}$, and the capacity of the low-rank MIMO channel for small SNR, $C_{low\_SNR}$, may expressed as:

$$C_{high\_SNR} \approx \min(N_R, M_T) \frac{P_T}{\sigma_n^2 M_T} N_R \tag{48}$$

$$C_{low\_SNR} \approx \frac{P_T}{\sigma_n^2} N_R \tag{49}$$

Altogether, from equation (22), (23), (33), (39),(41), (44), (47), (48), and (49) the MIMO channel and the MIMO-STBC channel capacity can be illustrated as two figures follow.

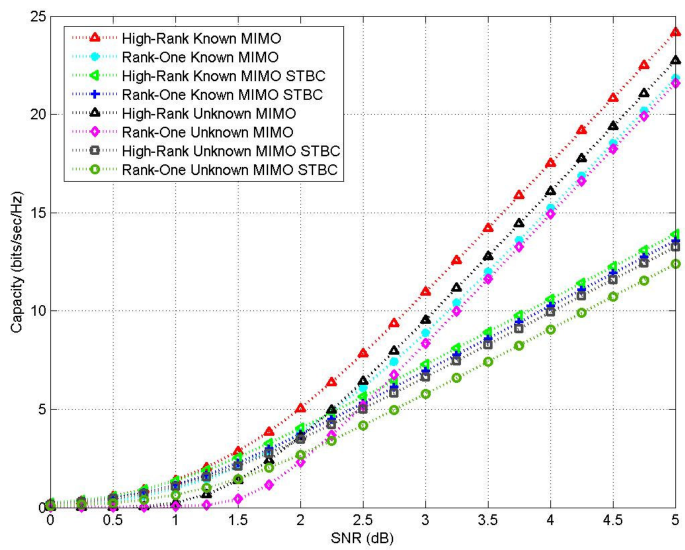


Figure 3. The capacity performance comparison between both of the MIMO systems and the MIMO-STBC systems with channel known and unknown, 2x2 antennas for high-rank and low-rank correlation

Figure 3 shows MIMO system and MIMO-STBC system capacity for **high-rank and low rank correlation** with the assumption of an average power constraint $P_{Tx}$, no channel knowledge at transmitter, **channel knowledge and no channel knowledge at the receiver** with 2x2, antennas of $T_X x R_X$ system. With the large S/N, in relation to Figure 4 that both of the MIMO systems and the MIMO-STBC systems with the no channel knowledge at the receiver have more capacity than the systems with the channel knowledge at the receiver. However with the small S/N , the systems with the channel knowledge at the receiver have better capacity performance than the other.

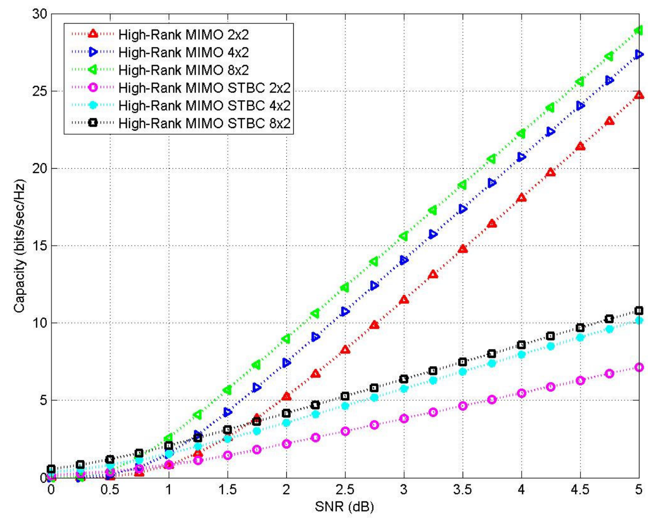


Figure 4. The capacity performance comparison between both of the MIMO systems and the MIMO-STBC systems with channel unknown, 2x2, 4x2, 8x2 antennas for high-rank correlation

Figure 4 shows MIMO system and MIMO-STBC system capacity for **high rank correlation** with the assumption of an average power constraint $P_{Tx}$, no channel knowledge at transmitter and **no channel knowledge at the receiver** with 2x2, 4x2, 8x2 antennas of $T_X x R_X$ system. (note : the ratio symbols of time slots of 4x2 and 8x2 antennas of $T_X x R_X$ system is ¾ ). With the same number of receive antennas and the large S/N, as shown in Figure 4 that for both of the MIMO channel and the MIMO-STBC channel, double of the number of transmit antennas has a large increase capacity performance. Generally, the MIMO channel capacity has better capacity performance than the MIMO-STBC channel. But in the small S/N, the MIMO channel capacity has bad capacity performance than the MIMO-STBC channel.

# 5 Conclusions

In general, both of the MIMO systems and the MIMO-STBC systems with the no channel knowledge at the receiver have more capacity than the systems with the channel knowledge at the receiver. However with the small S/N , the systems with the channel knowledge at the receiver have better capacity performance than the other.

So, for both of the MIMO channel and the MIMO-STBC channel, with the same number of receive antennas and the large S/N, double of the number of transmit antennas has a large increase capacity performance.

Therefore, with the same number of receive antennas and the large S/N, the MIMO channel has a better capacity performance than the MIMO with STBC in **high-rank and low rank correlation.** While in the small S/N, the MIMO channel capacity has bad capacity performance than the MIMO-STBC channel.

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RINA PUDJI ASTUTI: Ph D student at Department of Electrical Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia. E-mail: rina.p.a@plasa.com

TATI L.R MENGKO: Department of Electrical Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia. E-mail: tmengko@itb.ac.id

SUGIHARTONO: Department of Electrical Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia. E-mail: sugi_sugihartono@yahoo.com

ANDRIYAN B. SUKSMONO: Department of Electrical Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia. E-mail: suksmono@yahoo.com