# Finite-Length Analysis for Wireless Super-Dense Networks Exploiting Coded Random Access Over Rayleigh Fading Channels 

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#### Abstract

This paper presents finite-length performance analysis for wireless super-dense networks comprising two multiway relays (SDN-2MWR) to support full data-exchange among massive number of users/devices over Rayleigh fading channels. In practice, finite-length analysis is important since the networks should serve users/devices with low latency indicated by short number of time-slots. Due to the nature of massive number of users/devices, where scheduling over massive number of users (usually in hundred or thousand) is difficult, we exploit random access with a capability of error correction resembling low density parity check (LDPC) codes. In this paper, we show that the dynamic of Rayleigh fading is even beneficial to generate two independent graphs captured by the first and the second relay without requiring all users send messages independently to each relay. Independent graphs are essential in SDN-2MWR to ensure the probability of successful decoding as high as possible and to significantly reduce error-floor in finite-length setting. Based on the theoretical network capacity bound indicating the maximum achievable traffic supported by the networks, we found that for SDN-2MWR a significant gain closer to the bound with lower packet-loss-rate (compared to the dense network with a single relay) is achievable without assuming ideal independent graph even with simple degree distributions without irregularity.

Index Terms-Super-dense networks, Multiway relay, Rayleigh fading, coded random access, iterative successive interference cancellation (SIC).


## I. Introduction

Applications based on the Internet-of-things (IoT) supported by Machine-to-Machine (M2M) communication are expected to grow exponentially in the near future. Forecasts indicate that the number of connected things will reach about 50 billion for various application categories in 2020 [1]. This demand leads to the development of efficient wireless technologies serving very huge number of users or devices. In the rest of the paper, the terminologies of "user" and "device" are used interchangeably, except specified, since the future networks includes both traffic of human-to-human ( H 2 H ) and device-to-device (D2D).

As the nature of massive number of devices, random access technique is preferable since perfect scheduling can be

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Fig. 1. Super-dense network structures: (a) General super-dense network with a single multiway relay (SDN-1MWR) networks, (b) super-dense networ with 2 multiway relays (SDN-2MWR).
replaced by random scheduling, where all devices transmit their messages randomly without necessarily overhead of scheduling. This communication schemes are expected to open new grounds for designing good communications systems supporting super-dense networks.

In this paper, to serve full data-exchange which is typically required by IoT applications, we consider wireless superdense networks with the help of two multiway relays (SDN2MWR). Fig. 1(a) shows a general network structure of wireless super-dense network with a single multiway relay (SDN-1MWR), the potential applications of which are satellite systems, cellular networks and even disaster reponse networks for devastated areas of natural disasters. The useful analysis for SDN-1MWR networks can be found, for example, in [2] and recently in [3] when multipacket reception is considered.
It is, however, assumed in [2], [3] that the time-slot number is very large to satisfy the infinite length assumption in erasure probability and extrinsic information transfer (EXIT) analysis. With the same assumption of infinite length setting, the throughput of the networks can be improved by the help of new additional relay as proposed in [4] for two relays and [5] for general relay number.

This paper considers finite-length setting for networks serving massive number of users with the help of two multiway relays, called SDN-2MWR as shown in Fig. 1(b). We look
the challenges of the network performances with finite-length setting, while exploiting the dynamic of fading channels, to satisfy the future networks requirement on low latency services.
The analysis of finite-length setting has been presented, for example, in [6], where the performance is tested on the regular degree distribution. The analysis of finite-length setting based on irregular degree distribution followed by stopping set analysis was studied in [7]. However, none of them is assuming Rayleigh fading channels and its affect to the performance of super-dense networks, where diversity is usually required to compensate deep fading channels.
The analysis of diversity is addressed in, for example, [8], where the diversity is also achieved with additional relays. It is assumed in [6], [8] that the systems are for many-toone applications, however, in this paper, we consider many-to-many applications.

We summarize our contributions as follows: (i) we analyze the induced distribution caused by the Rayleigh fading channels, and (ii) based on some important stopping sets, we derive finite-length performance analysis for wireless superdense networks SDN-2MWR under Rayleigh fading channels.

## II. System Model

We consider a system model as shown in Fig. 1(b), where two relays are located fairly such that all users can reach the both of them. To ensure low computational complexity, amplify-and-foward (AF) protocol is considered, which is also well suited for a large number of users since the relay does not need to decode messages of all users.

Two types of encoding schemes, i.e., physical encoding and network encoding are involved. To simplify the analysis, physical encoding is assumed to be perfectly performed by using simple and reliable coding schemes, e.g., memory-1 convolutional codes followed by a doped-accumulator [3], [9], which is efficient enough over Rayleigh fading channels.

The channel between user and relay is assumed to be suffering from block Rayleigh fading, where the channel gains are assumed to be perfectly known at both of the relays and the users, e.g., by sending the channel state information in a header of every packet. We also assume the receive signal-to-noise power ratio (SNR) is beyond a threshold, which is around 3 dB in [3], such that all messages are correctly decodable physically. Other parameters are assumed by following the model of dense networks in [4], [5].
We consider sd-MWMR networks serving $(M+1)$ users, where the information is exchanged within two phases, i.e., multiple access channel (MAC) and broadcast (BC) phases. In MAC phase all users transmit/broadcast their messages to both the relays randomly without coordination, while in the $B C$ phase two relays broadcast the packets ${ }^{1}$ to all users.
An example of received packets during MAC and BC phases of a system having 5 users are shown in Figs. 2(a), (b), and (c),

[^0]| $R_{1}$ | PTS $T_{1}$ |  |  | PTS $T_{2}$ |  |  | PTS $T_{3}$ |  |  | PTS $T_{4}$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 |  |
| $u_{1}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{2}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |$u_{3}$

(a)
faded

| $R$ | PTS $T_{1}$ |  |  | PTS $T_{2}$ |  |  | PTS $T_{3}$ |  |  | PTS $T_{4}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 |
| $u_{1}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{2}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{3}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{4}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{5}$ |  |  |  |  |  |  |  |  |  |  |  |  |
|  | non- | ade |  |  |  | (b) |  |  |  |  | fa | ded |


|  | PTS $T_{1}$ |  |  | PTS $T_{2}$ |  |  | PTS $T_{3}$ |  |  | PTS $T_{4}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 | MAC | BC1 | BC2 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{2}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{3}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{4}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| $u_{5}$ |  |  |  |  |  |  |  |  |  |  |  |  |

(c)


Fig. 2. Users' transmissions within one frame containing several pair-of-timeslots (PTSs): (a) at Relay $R_{1}$, (b) at Relay $R_{2}$, (c) at user $u_{1}$, and (d) the bipartite graph of received packets at $u_{1}$.
where the packets may be received or erased depending on the fading channels. Fig. 2(a) shows that packets from users $u_{1}, u_{2}$ and $u_{3}$ are all erased during the MAC phase at PTS $T_{2}$ at relay $R_{1}$, but correctly received at relay $R_{2}$. It is also important to note here that we assume BC phases are not suffering from severe fading channels to simplify the analysis. ${ }^{2}$

We define pair-of-time slot (PTS) as [2], [3] comprising MAC, BC1, and BC2 as shown in Fig. 2. The $t$-th PTS is defined as $T_{t}=(M A C, B C 1, B C 2)_{t}$, where in this paper

[^1]the length of PTS $T_{t}$ is set to be finite at around 50 to avoid long delay of full data-exchange.

Fig. 2(d) shows the equivalent bipartite graph of messages received by user $u_{1}$ in Fig. 2(c) after self-interference subtraction. User $u_{1}$ detects the packet from other 4 users ( $u_{2}, u_{3}, u_{4}, u_{5}$ ) after performing self-interference subtraction with its own message. The circles represent user nodes having degree $h$, while the squares represent PTS nodes having degree $d$. Please note that we can control the degree $h$, but not for the degree $d$.

One frame comprises $N$ PTS, of which the offered traffic seen by every user (except the sender itself) is then defined as [10]

$$
\begin{equation*}
G=\frac{k M}{N} \tag{1}
\end{equation*}
$$

where $k$ is a parameter depending on the network encoding scheme, which is set to $k=1$ in this paper for repetition codes. Each user randomly picks a code $c_{h}=(h, 1)$ from network code sets $\mathcal{C}=\left\{c_{2}, \ldots, c_{n}\right\}$ according to a degree distribution $\Lambda=\left\{\Lambda_{2}, \Lambda_{3}, \ldots, \Lambda_{n}\right\}, \sum_{h=2}^{n} \Lambda_{h}=1,{ }^{3}$ where $\Lambda_{h}$ is the probability of any user nodes having $h$ edges. A code $c_{h}$ is a linear code with length $h$ with rate $R_{h}=\frac{1}{h}$. Given $\Lambda$, the average rate of the network, seen by each user, is [10]

$$
\begin{equation*}
\bar{R}_{o}=\frac{1}{\bar{n}}=\frac{1}{\sum_{h=2}^{n} \Lambda_{h} h} \tag{2}
\end{equation*}
$$

## III. Decoding Strategy

The bipartite graph shown in Fig. 2(d) represents the SDN2WMR networks resembling the structure of two connected low density parity check (LDPC) codes, which is redrawn in Fig. 3(a) as graph $\mathcal{G}=\left(\mathcal{G}_{R_{1}}, \mathcal{G}_{R_{1}}, \mathcal{E}^{e}\right)$ of two connected bipartite graphs $\mathcal{G}_{R_{1}}$ and $\mathcal{G}_{R_{2}}$ via external links $\mathcal{E}^{e}$. The top bipartite graph $\mathcal{G}_{R_{1}}=\left(\mathcal{U}, \mathcal{S}_{1}, \mathcal{E}_{1}^{i}\right)$ is a graph comprising user nodes $\mathcal{U}=\left\{u_{2}, \ldots, u_{5}\right\}$, slot nodes $\mathcal{S}_{1}=\left\{T_{1}, T_{3}, \ldots, T_{5}\right\}$ and internal edge $\mathcal{E}_{1}^{i}$ for messages received via the relay $R_{1}$. Similarly, the lower graph $\mathcal{G}_{R_{2}}=\left(\mathcal{U}, \mathcal{S}_{2}, \mathcal{E}_{2}^{i}\right)$ is a bipartite graph for the messages received via relay $R_{2}$.
The original degree distribution of user nodes is expressed by polynomial

$$
\begin{equation*}
\Lambda(x)=\sum_{h=2}^{H} \Lambda_{h} x^{h} \tag{3}
\end{equation*}
$$

After the fading channels, the degree distribution is induced to

$$
\begin{equation*}
\tilde{\Lambda}(x)=\sum_{h=0}^{H} \Lambda_{h} x^{h} \tag{4}
\end{equation*}
$$

where degrees $h=0$ and $h=1$ causing error floor are unavoidable. The induced degree distribution at $\mathcal{G}_{R_{1}}$ and $\mathcal{G}_{R_{2}}$ shown in Fig. 3(a) are $\tilde{\Lambda}_{R_{1}}(x)=\frac{1}{2} x+\frac{1}{2} x^{2}, \tilde{\Lambda}_{R_{2}}(x)=$ $\frac{1}{4} x+\frac{1}{2} x^{2}+\frac{1}{4} x^{3}$, respectively. The second PTS of $\mathcal{G}_{R_{1}}$ is erased because of a deep fading, but not for the PTS of $T_{2}$ of $\mathcal{G}_{R_{2}}$. It is an example showing how the dynamic of fading channel can be exploited to generate two different graphs.

[^2]

Fig. 3. Steps of iterative decoding between graphs $\mathcal{G}_{R 1}$ and $\mathcal{G}_{R 2}$ avoiding decoding stuck in each graph.

We also define user node degree distribution from the edgeperspective as

$$
\begin{equation*}
\lambda(x)=\sum_{h=0}^{H} \lambda_{h} x^{h-1} \tag{5}
\end{equation*}
$$

where $\lambda_{h}=\frac{h \Lambda_{h}}{\sum_{h} h \Lambda}$.
For the slot ${ }^{n}$ node degree distribution, because the transmission is random, the distribution of slot nodes is following Poisson distribution as

$$
\begin{equation*}
\Psi(x)=\sum_{d=0}^{M} \Psi_{d} x^{d} \approx e^{-\frac{G}{R}(1-x)}, \tag{6}
\end{equation*}
$$

of which the edge-perspective degree distribution $\rho(x)=$ $\Psi(x)$.
From this point, our analysis of dense networks is based on erasure channels, because we assume that the packets are erased when they are collided. Let assume that a message $s_{i}$ is transmitted from user $u_{i}$. In Fig. 3(a) of graph $\mathcal{G}_{R_{1}}$, a message $s_{2}$ at PTS $T_{1}$ is received clearly, while at PTS $T_{3}$, the message $s_{2}, s_{3}$ and $s_{4}$ are collided, where the received signal is $s_{2}+s_{3}+s_{4}$.
Let assume that $p$ as an erasure probability of an edge going out from a slot node and $q$ from a user node indication packets are unresolved. Since we have two connected bipartite graphs
$\mathcal{G}_{R_{1}}, \mathcal{G}_{R_{2}}$, we define an external erasure probability $p^{\text {ext }}$ and $q^{e x t}$ between two graphs as shown in Fig. 3(a).

Slot nodes are unaffected by the relay number, therefore, the erasure probability from a slot node following (6) is [11]

$$
\begin{equation*}
p=1-\exp \left\{-q \frac{G}{\bar{R}_{n}}\right\} \tag{7}
\end{equation*}
$$

The exponential function in the $p$ is coming from the fact of random transmission that results in the distribution of degree $d$ of the slot nodes allowing Poisson distribution. This assumption is correct when the $(M+1)$ users and $N$ PTS are very large, but keeping throughput $G$ constant.

Beside the repetition codes, when the network code rates $\bar{R}_{n}>\frac{1}{2}$ are the target of the network design, the maximum distance separable (MDS codes may be considered. The following subsections discuss the erasure probability of the codes.

## A. Repetition Codes

With the existing of the second relay, the internal erasure probabilities is

$$
\begin{equation*}
q=\sum_{h=2}^{n_{c}} \lambda_{h}(p)^{h-1} p^{e x t} \tag{8}
\end{equation*}
$$

where $\lambda_{h}$ is the degree of user nodes from edge perspective [2], [3]. For external $q^{e x t}$, as indicated by Fig. 3(a), the erasure probability is

$$
\begin{equation*}
q^{e x t}=\sum_{h=2}^{n_{c}} \lambda_{h}(p)^{h} \tag{9}
\end{equation*}
$$

## B. MDS Codes

For an $\left(n_{h}, k\right)$ MDS code, the internal erasure probability is expressed as

$$
\begin{equation*}
q=\sum_{\ell=0}^{k-1}\binom{n_{h}-1}{\ell}(1-p)^{\ell} p^{n_{h}-\ell-1} p^{e x t}+p^{n_{h}-1}\left(1-p^{e x t}\right) \tag{10}
\end{equation*}
$$

and for external erasure probability as

$$
\begin{equation*}
q^{e x t}=\sum_{\ell=0}^{k-1}\binom{n_{h}}{\ell}(1-p)^{\ell} p^{n_{h}-\ell-1} \tag{11}
\end{equation*}
$$

Some discussions on erasure probability of MDS codes can be seen, for example, in [10].

## C. Decoding Algorithm

Given the erasure probabilities as described above, this paper proposes a simple joint decoding technique for a big graph involving two multiway relays. To simplify the algorithm, we consider an example of decoding steps shown in Fig. 3 for user $u_{1}$.

Because two relays are involved, the decoding speed is improved since when a decoding process get stuck in one graph, the process is then moved to another graph, which may not get stuck because of unequal graph. The unequal graph is possible since the signals received by the relay $R_{1}$ and $R_{2}$ are experiencing different fading channels. This process also reduce the error-floor introduced by both the Rayleigh fading channels and the stopping sets.

```
Algorithm 1: Joint Decoding of Networks with SI.
    Data: Connected bipartite graphs \(\mathcal{G}_{R}\) and \(\mathcal{G}_{D}\).
    Result: Messages from all user nodes are decoded.;
    Initialization: Perform self-cancellation on graph \(\mathcal{G}_{R}\) and \(\mathcal{G}_{D}\).
    while All users are not decoded or maximum iterations is not
    reached do
        Loop \(\mathcal{G}_{D}\) : Consider \(\mathcal{G}_{D}\) and find user \(u_{i}\) connected to a
        slot node having degree \(d\).;
        if Degree \(d=1\) is found then
            Decode message of \(u_{i}\) and subtract the received signals
            in all slot nodes connected to user \(u_{i}\) with user \(u_{i}\) 's
            own message as shown in Figs. 3(a) and (d);
        else
            Go to graph \(\mathcal{G}_{R}\) and perform Loop \(\mathcal{G}_{R}\);
        end
        Loop \(\mathcal{G}_{R}\) : Consider \(\mathcal{G}_{R}\) and find a user \(u_{j}\) connected to a
        slot node having degree \(d\).;
        if Degree \(d=1\) is found then
            Subtract the received signals at all slot nodes
            connected to user \(u_{i}\) with user \(u_{i}\) 's message as shown
            in Figs. 3(b) and (c);
        else
            Go to graph \(\mathcal{G}_{D}\) and perform Loop \(\mathcal{G}_{D}\);
        end
    end
```


## IV. Network Capacity Bound

The network capacity bound we consider in this paper is a bound indication the maximum number of users (expressed in $G$ ) can be resolved given the rate $\bar{R}$. We refer to the network capacity bound derived in [5], where detailed derivation is described for general number of relays. For the case of two relays considered in this paper, the bound is expressed as

$$
\begin{equation*}
\frac{k \bar{R}}{k+\bar{R}}+\left(\frac{\bar{R}}{G}\right) e^{-\frac{G}{\bar{R}}}-\frac{\bar{R}}{G}<0 \tag{12}
\end{equation*}
$$

of which the curve is shown in Fig. 4.
It is confirmed from Fig. 4 that the network capacity bound of two multiway relays is higher than the original single relay bound. This a good news since new bound can minimize the physically impossible region. However, for $\bar{R}>\frac{1}{2}$, the sudden dropped lines $\mathrm{AB}, \mathrm{CD}$, and the rest, happens because of nonsmooth change of the network coding rates $\bar{R}_{n}=k /(k+1)$ with $k=\{1,2, \ldots, 500\}$, especially when $k$ is small.
We have performed several tests for some degree distributions to verify the accuracy of the new bound of (12). We use $(k, 1)$ repetition codes of $\Lambda_{c}=x^{k}$ and $(k+1, k)$ MDS codes $\Lambda_{d}$ with $k=\{1,2, \ldots, 10\}$. It is shown by Fig. 4 that the new bound is confirmed to be accurate both by repetition codes and MDS codes. The repetition codes achieve maximum $G$ only when $R_{n}=1 / 2$, while the $(k+1, k)$ MDS codes achieve the bound almost at every point.

It should be noted here that in the following sections, we consider only repetition codes, due to the limited space, with the following degree distributions

$$
\begin{align*}
& \Lambda_{a}(x)=x^{2}  \tag{13}\\
& \Lambda_{b}(x)=x^{3}  \tag{14}\\
& \Lambda_{c}(x)=0.87 x^{3}+0.13 x^{8} \tag{15}
\end{align*}
$$



Fig. 4. The new theoretical bound gained from the second multiway relay.

## V. Theoretical Performances: Induced Distribution and Stopping Sets

The error-floor in performance is unavoidable when fading channels are considered. This section analyzes the induced distribution due to fading effects. The second reason of errorfloor is represented by the stopping sets, the probability of which is high in finite-length setting.

## A. Induced Distributions

When the users are mobile, the channels between users and the relay are following the Rayleigh distribution with the probability

$$
\begin{equation*}
\operatorname{Pr}(\gamma)=\frac{1}{\Gamma} \exp \left(\frac{\gamma}{\Gamma}\right) \tag{16}
\end{equation*}
$$

with $\Gamma$ being the average received power. We assume that there is a threshold $P_{T}$ such that when a message with a power below $P_{T}$ is regarded as a non-received message or erased. ${ }^{4}$ In bipartite graph connecting users and PTS nodes, an edge representing this situation can be deleted. The probability of an edge to be deleted or dropped due to Rayleigh fading channels is therefore defined as

$$
\begin{equation*}
\left.\operatorname{Pr}\left(\gamma \leq P_{T}\right)=\int_{0}^{P_{T}} \operatorname{Pr}(\gamma)\right) d \gamma=1-\exp \left(-\frac{P_{T}}{\Gamma}\right) \tag{17}
\end{equation*}
$$

This situation makes a translation of degree distribution from the original $\Lambda(x)$ to a new induced distribution $\tilde{\Lambda}(x)$, which is important to predict the performance of the systems over Rayleigh fading channels.

[^3]TABLE I
AN EXAMPLE OF INDUCED DISTRIBUTION FROM
$\Lambda_{c}(x)=0.87 x^{3}+0.13 x^{8}$ то $\tilde{\Lambda}_{c}(x)=0.01+0.1 x+0.35 x^{2}+$ $0.41 x^{3}+0.01 x^{4}+0.02 x^{5}+0.04 x^{6}+0.04 x^{7}+0.02 x^{8}$.

|  | $x^{2}$ | $x^{3}$ | $x^{8}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0.87 | 0.13 |  |
| $x^{0}$ | 0.05 | 0.01 | 0 | 0.01 |
| $x^{1}$ | 0.34 | 0.11 | 0 | 0.10 |
| $x^{2}$ | 0.61 | 0.40 | 0 | 0.35 |
| $x^{3}$ |  | 0.47 | 0.01 | 0.41 |
| $x^{4}$ |  |  | 0.06 | 0.01 |
| $x^{5}$ |  |  | 0.17 | 0.02 |
| $x^{6}$ |  |  | 0.31 | 0.04 |
| $x^{7}$ |  |  | 0.31 | 0.04 |
| $x^{8}$ |  |  | 0.14 | 0.02 |

Based on (17), the induced probability of a user with a degree $h$ to a user with a degree $\tilde{h}$, where $0 \leq \tilde{h} \leq h$ is

$$
\begin{equation*}
P_{h \rightarrow \tilde{h}}=\binom{h}{h-\tilde{h}}\left(P_{d}\right)^{h-\tilde{h}}\left(1-P_{d}\right)^{\tilde{h}}, \tag{18}
\end{equation*}
$$

where $P_{d}=1-\exp \left(\frac{P_{T}}{\Gamma}\right)$ is the probability of an edge dropped due to Rayleigh fading channels as defined in (17). An example of induced distribution is shown in Table. I for sd-MWMR networks with $\Lambda_{c}(x)=0.87 x^{3}+0.13 x^{8}$. The induced distribution in general is expressed by

$$
\begin{equation*}
\tilde{\Lambda}(x)=\sum_{\tilde{h}=0}^{H}\left(\sum_{h=0}^{H} \Lambda_{h} P_{h \rightarrow \tilde{h}}\right) x^{\tilde{h}} \tag{19}
\end{equation*}
$$

where $H$ is the maximum degree of the user nodes. The biggest contribution to the error-floor from the induced distribution is users with degree $h=0$ and degree $h=1$ since the existing $P_{h \rightarrow 0}$ and $P_{h \rightarrow 1}$ caused by fading channels.

## B. Important Stopping Sets

Stopping set is a harmful structure, where no other ways for some users trapped in set to be decoded. An example of stopping set is shown in Fig. 5(a), where two degree $h=2$ users transmit their messages at the same PTS. Stopping sets involving three and four users are shown in Figs. 5(b)-(c). It is important to note here that the stopping set occurs if the users of both relays are experiencing stopping sets.

Based on the network capacity bound shown in (12), the most beneficial rate for sd-MWMR with two relays is $R_{n}=$ $1 / 2$, the most favorable degree of which is $h=2$, since it provides highest offered traffic at around $G=1.4$ packet/slot. Therefore, we consider stopping sets shown in Fig. 5 as the important stopping sets.
Based on [7], the probability appearance of stopping sets $\rho(\mathcal{S S})$ increases with the decrease of time slots in finite-length

(a). $\mathcal{S S}_{1}$

(b). $\mathcal{S S}_{2}$

(c). $\mathcal{S S}_{3}$

Fig. 5. Important stopping sets: (a). involving two users with $\nu\left(\mathcal{S S}_{1}\right)=[0,0,2,0,0]$, (b). three users $\nu\left(\mathcal{S S}_{2}\right)=[0,0,3,0,0]$, and (c). four users with $\nu\left(\mathcal{S S}_{3}\right)=[0,0,4,0,0]$.
setting. The occuring probability of stopping sets $\mathcal{S S}_{1}, \mathcal{S S}_{2}$ and $\mathcal{S} \mathcal{S}_{3}$ shown in Fig. 5, only in one graph, is expressed as,

$$
\begin{align*}
\rho\left(\mathcal{S S}_{1}\right) & =\frac{M!}{(M-2)!} \frac{\Lambda_{2}{ }^{2}}{2!} \frac{2}{(N-1) N}  \tag{20}\\
\rho\left(\mathcal{S S}_{2}\right) & =\frac{M!}{(M-3)!} \frac{\Lambda_{2}{ }^{3}}{3!} \frac{8(N-2)}{(N-1)^{2} N^{2}}  \tag{21}\\
\rho\left(\mathcal{S S}_{3}\right) & =\frac{M!}{(M-4)!} \frac{\Lambda_{2}}{4!} \frac{288(N-3)(N-2)}{3(N-1)^{3} N^{3}} \tag{22}
\end{align*}
$$

Since the stopping set occurs only when the users of both graph experience the same stopping set, the probability $\rho(\mathcal{S S})$ decreases to

$$
\begin{align*}
& \tilde{\rho}\left(\mathcal{S S}_{1}\right)=\rho\left(\mathcal{S} \mathcal{S}_{1}\right)^{2} \cdot \frac{1}{\binom{M}{2}},  \tag{23}\\
& \tilde{\rho}\left(\mathcal{S S}_{2}\right)=\rho\left(\mathcal{S} \mathcal{S}_{2}\right)^{2} \cdot \frac{1}{\binom{M}{3}},  \tag{24}\\
& \tilde{\rho}\left(\mathcal{S S}_{2}\right)=\rho\left(\mathcal{S S}_{3}\right)^{2} \cdot \frac{1}{\binom{M}{4}} . \tag{25}
\end{align*}
$$

The PLR for degree $h=2$ can be approximated as

$$
\begin{equation*}
P_{2} \approx \frac{2}{M \Lambda_{2}}\left\{\tilde{\rho}\left(\mathcal{S} \mathcal{S}_{1}\right)+\tilde{\rho}\left(\mathcal{S} \mathcal{S}_{2}+\tilde{\rho}\left(\mathcal{S S}_{3}\right)\right\}\right. \tag{26}
\end{equation*}
$$

When other degrees $h$ are considered, the PLR are taken by averaging as

$$
\begin{equation*}
P_{t o t}=\sum_{h=0}^{H} \Lambda_{h} P_{h} \tag{27}
\end{equation*}
$$

Similar to the stopping set, users degree-0 due to deep fading are also unresolved. The probability of users with degree-0 is

$$
\begin{equation*}
\rho\left(\mathcal{S S}_{0}\right)=M \Lambda_{0} \tag{28}
\end{equation*}
$$

and decreases to

$$
\begin{equation*}
\tilde{\rho}\left(\mathcal{S S}_{0}\right)=\rho\left(\mathcal{S} \mathcal{S}_{0}\right)^{2} \cdot \frac{1}{\binom{M}{1}} \tag{29}
\end{equation*}
$$

in the case of two relays. Equation (29) indicates that even the degree- 0 probability can be significantly reduced by the help of the second relay.


Fig. 6. PLR Performances under block Rayleigh fading channels with PTS $N=50$.

## VI. Performance Evaluations

The performances of the decoding scheme is evaluated via computer simulations for $N=50$ and $N=200$ under block Rayleigh fading channels. The analysis on fading channels is very important, because in practice $\mathcal{G}_{R_{1}} \neq \mathcal{G}_{R_{2}}$ is achievable only with fading channels to exploit the nature of broadcast messages received by the both relays.

An edge is dropped from the graph when the received power

$$
\begin{equation*}
P_{r}=\left|h_{f}\right|^{2} \leq P_{T}, \tag{30}
\end{equation*}
$$

with $h_{f}$ is the Rayleigh fading gain between each users and the relays, and $P_{T}$ being the threshold of received power. In this simulation, we set $P_{T}=0.25$, as an example. ${ }^{5}$ The higher $P_{T}$, the better the difference of two graphs that results in better throughput performances. However, the higher $P_{T}$ potentially causes serious error-floor because many deleted edges in $\mathcal{G}_{R_{1}}$ and $\mathcal{G}_{R_{2}}$ that results in higher degree $h=0$.
Figs. 6 and 8 show the simulation results under the dynamic block Rayleigh fading channels. It is shown that $\Lambda_{a}(x)$ has the best throughput, which is predicted by the theoretical network capacity bound in (12). Fig. 6 also confirms the accuracy of the theoretical performance analysis for short $N$. For $G=$ 1 packet/slot, the second relay helps to decrease the PLR from $P_{t o t}=0.5$ down to $P_{t o t}=0.0001$ with $\lambda_{b}(x)=x^{3}$. Fig. 7 compares the throghput of the conventional slotted ALOHA with two relays, which is confirmed still below 1 packet/slot under Rayleigh fading channels.
Fig. 8 evaluates the performance of systems with (i) longer block-lentgh, and (ii) irregular code distributions. The results indicated that under fading channel, larger block-length does

[^4]

Fig. 7. PLR Performances under block Rayleigh fading channels with $P_{T}=0.25$.
not help much, because the performance is dominated by the degrees $h=0$ or $h=1$. Fig. 8 shows that $\Lambda_{c}(x)=0.87 x^{3}+$ $0.13 x^{8}$, which was the best distribution in [7] exhibiting lowest error-floor, has similar error-floor as simple code distribution $\Lambda_{b}(x)=x^{3}$. It confirms that irregularity in coding distribution also do not help much in fading channels. Further optimization considering induced distribution may be needed for superdense network suffering from block Rayleigh fading channels.

## VII. Conclusions

We have presented finite-length performance analysis for wireless super-dense networks with multiway multirelay (SDN-2MWR) over Rayleigh fading channels. The dynamics of the fading channels are exploited to obtain two independent graphs captured by each relay. We proposed iterative interference cancellation over a sparse graph involving two multiway relays with finite-length setting; furthermore, we confirmed the results using the theoretical network capacity bound as a bound indicating the maximum achievable offered traffic. We found that two relays help both on (i) the improvement of throughput performances, and (ii) decrease significantly the error-floor, which is highly required for future wireless dense networks. We also found that regular degree distributions are good enough in multiple relay case. Further investigation on the induced degree distribution may be needed as well as the detection of a packet to be erased in a time-slot.

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Fig. 8. PLR Performances comparison simple distribution $\Lambda_{b}(x)=x^{3}$ with irregular code distribution, $\Lambda_{c}(x)=0.87 x^{3}+0.13 x^{8}$, under block Rayleigh fading channels with PTS $N=50$ and $N=200$ time-slots.
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[^0]:    ${ }^{1}$ Terminologies of "messages", "packets" and "information" may be used interchangeably except specified.

[^1]:    ${ }^{2}$ The analysis of two cascaded fading channels is left for future study.

[^2]:    ${ }^{3}$ To obtain a stable network, it should be kept that $h \geq 2$.

[^3]:    ${ }^{4}$ In practice, we assume that there is a mechanism like a CRC to check whether a message or packet supposed to receive in a certain PTS has been completely received or completely erased.

[^4]:    ${ }^{5}$ More practical values of $P_{T}$ might be needed based on experiment.

